EEC 4230 - Mobile Communication Systems

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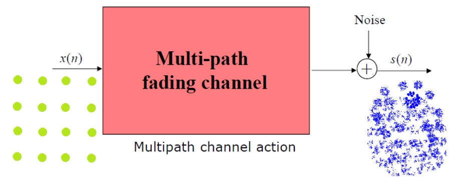
Lecture 9: BER Performance and Capacity of Mobile Radio-II

Outline

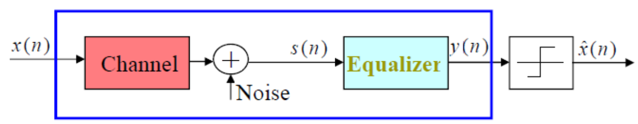
1. Capacity of fading channels
2. Digital modulation performance in fading channels
3. Equalization
4. Diversity
5. Channel coding techniques for mobile radio

Equalization

- In Digital transmission, we denote the transmitted signal as $x(n)$ and not x(t).
- Equalizer is needed when the channel introduces ISI (due to multipath fading).



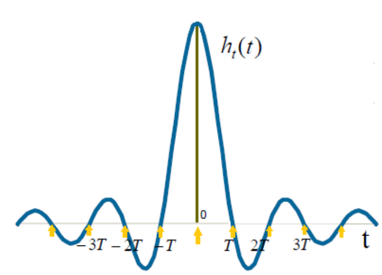
- Equalizer removes ISI introduced by the channel and It must be adaptive since the channel is generally unknown and time-varying.



- Overall impulse response of the system: $h_t(t) = h_{ch}(t) * h_{eq}(t)$
- Thus, output of equalizer: $y(n) = y(nT_s) = \sum_{k=-\infty}^{\infty} h_t(kT_s)x((n-k)T_s)$

Nyquist criterion for ISI cancellation

- Nyquist observed that the effect of channel ISI could be completely removed if is designed such that at every sampling instant (when output of equalizer is taken), the response due to all interfering symbols are zero, except the current (desired) symbol, i.e if $h_t = \begin{cases} K & \text{if } n = 0 \\ 0 & \text{if } n \neq 0 \end{cases}$.
- Thus $h_t(t) = \frac{\sin(\pi t/T_s)}{\pi t/T_s} = \text{sinc}(t/T_s)$
- Which gives flat spectrum in frequency domain.



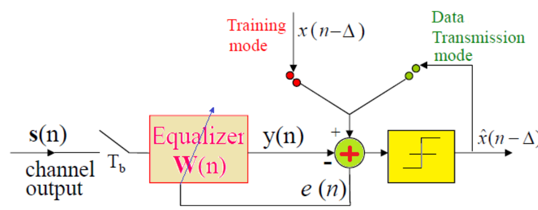
Adaptive Algorithms:

- Equalizer employs an adaptive algorithm to adjust its coefficients iteratively, to minimize the error between transmitted sequence and equalizer output.
- It tracks the channel impulse response h_{ch} to 'finds' (or 'adjusts') h_{eq} such that $h_t = h_{ch} * h_{eq}$ satisfies the Nyquist criterion at every time instant.
- When this is achieved exactly, the sampled output of the Equalizer is ISI-free.

Equalization

Training mode: Period during which equalizer coefficients are adjusted until the optimum values that minimizes the error $e(n)$ is obtained. This is done prior to data transmission (some processing delay introduced). During training, a sequence of training symbols, known to both the TX and RX are used.

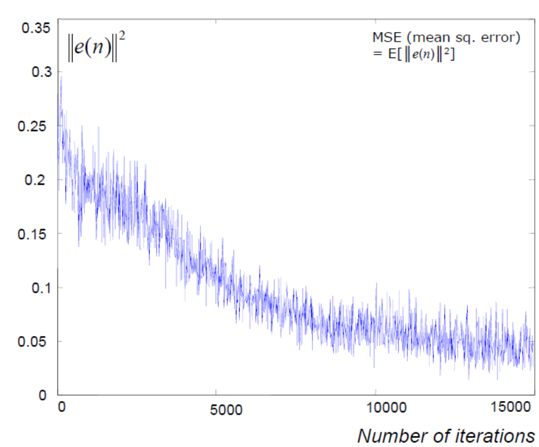
Decision-directed (transmission) mode: The optimum equalizer coefficients obtained during the training mode are used to equalize the channel during data transmission mode. Periodic update or re-training of the equalizer may have to be done as necessary.



$W(n)$ models $h_{eq}(t)$
Since channel is not fixed, $W(n)$ is constantly being adjusted during data transmission mode.

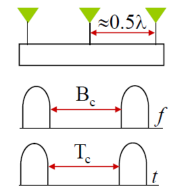
Least mean squared error (LMS) Algorithm

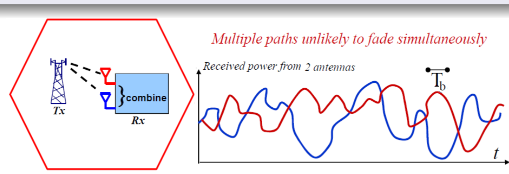
- It is the simplest and the most widely used adaptation algorithm developed by Widrow et all (1975).
- Equalizer coefficient is modeled as an FIR filter.
  $y(n) = \sum_{i=0}^{L-1} w_i s(n-i)$
  $e(n) = x(n-\Delta) - y(n)$
  $w_i(n+1) = w_i(n) - \mu \frac{\partial |e(n)|^2}{\partial w_i(n)}$
  $w_i(n+1) = w_i(n) + 2\mu e(n) s(n-i)$
- μ: the convergence coefficient.
- Other more powerful but more complex adaptive algorithms include the recursive least square (RLS) algorithm.



Diversity Techniques

- Diversity means sending same bits over independent fading paths, and combine the received signal to mitigate fading effects
- The probability that received signal powers from all M antennas fade below a critical value γ is p^M, where p is the probability that any one path will fade below γ, resulting in BER improvement.



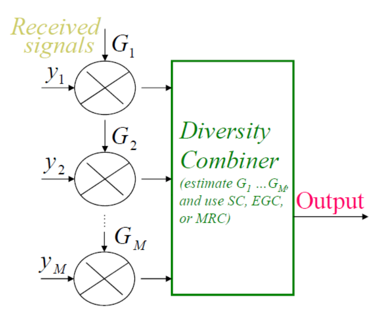


- Space (Antenna) Diversity: Multiple antenna elements separated by de-correlation distance. It is widely deployed in wireless systems.
- Polarization Diversity: When two transmit or receive antennas with different polarizations are used. This method provides 2-branch diversity only.
- Frequency Diversity: Multiple narrow band separated by coherence bandwidth, Bc.
- Time Diversity: Multiple time slots separated by channel coherence time, Tc.
- Most 2.5/3G cellular systems have installed 2-branch antenna diversity at the BS.

Space (Antenna) Diversity

Traditional Combining Techniques

- Selection combining (SC): The diversity branch with the highest instantaneous SNR is selected and used for signal detection.
- Equal-Gain combining (EGC): All branches are summed up, with equal weighting, and the composite signal is used.
- Maximal-Ratio combining (MRC): All paths are summed up, but each branch is weighted with the respective branch gain (optimal weighting) in order to maximize combiner output SNR.



- Adv/Disadvantages: SC is simple but provides the least BER improvements. MRC provides the best BER improvement but complex. EGC is in between these two.
- Other techniques: Generalized selection (GSC) and threshold-based SNR combiner.

Combiner output = $G_1 y_1(t) + G_2 y_2(t) + \ldots + G_M y_M(t)$

For SC: set $G_i = \begin{cases} 1 & \text{for } \max(y_1, y_2, \ldots, y_i, \ldots, y_M) \\ 0 & O.W. \end{cases}$

For EGC: set $G_1 = G_2, \ldots = G_M = 1$.

For MRC: use respective G_i.

Selection Combining (SC)

- Consider M independent Rayleigh fading branches, each with equal average SNR, γ_0.
- Let γ_i be the SNR in each branch, then γ_i is exponentially distributed with pdf
  $p(\gamma_i) = \frac{1}{\gamma_0} e^{-\frac{\gamma_i}{\gamma_0}}, \qquad i = 1, ..., M, \gamma_i \geq 0.$
- The probability that the instantaneous SNR at any branch will be less than some threshold γ is: $p(\gamma_i \leq \gamma) = \int_0^{\gamma} p(\gamma_i) d\gamma_i = 1 - e^{-\frac{\gamma}{\gamma_0}}$ (i.e. No diversity Case)
- The probability that all the branches would simultaneously have SNR less than γ is
  $p(\gamma_i, ..., \gamma_M \leq \gamma) = p(\gamma_1 \leq \gamma)....p(\gamma_M \leq \gamma) = [1 - e^{-\frac{\gamma}{\gamma_0}}]^M$ (i.e. SC diversity improvement)
- The SNR improvement for SC is given by: $\bar{\gamma}_{SC} = \gamma_0 \sum_{k=1}^{M} \frac{1}{k}$ (has a proof)

Proof of Selection combining (SC) improvement

- CDF of γ_{SC} is $p(\gamma_i, ..., \gamma_M \leq \gamma) = [1 - e^{-\frac{\gamma}{\gamma_0}}]^M$
- pdf of γ_{SC} is : $f_{\gamma_{SC}}(\gamma) = \frac{d}{d\gamma} CDF = M(\frac{1}{\gamma_0}) e^{-\frac{\gamma}{\gamma_0}} [1 - e^{-\frac{\gamma}{\gamma_0}}]^{M-1}$
- Thus, average SNR of SC is $\bar{\gamma}_{SC} = \int_0^{\infty} \gamma f_{\gamma_{SC}}(\gamma) d\gamma$

Space (Antenna) Diversity

Maximum Ratio combining (MRC)

- Consider M independent Rayleigh fading branches, each with equal average SNR, γ_0.
- For MRC, the pdf of combined SNR is the pdf of the sum of M Gaussian r.v. = Chi-square distribution and is given by:
  $p(\gamma_{MRC}) = \frac{\gamma_{MRC}^{M-1} e^{-\frac{\gamma_{MRC}}{\gamma_0}}}{\gamma_0^M (M-1)!}, \qquad \gamma_{MRC} \geq 0, \gamma_{MRC} = \sum_{i=1}^{M} \gamma_i$
- The probability that γ_{MRC} is less than γ is
  $p(\gamma_{MRC} \leq \gamma) = \int_0^{\gamma} p(\gamma_{MRC}) d\gamma_{MRC} = 1 - e^{-\frac{\gamma}{\gamma_0}} \sum_{k=1}^{M} \frac{\frac{\gamma}{\gamma_0}^{k-1}}{(k-1)!}$ (i.e. MRC diversity improvement)
- The SNR improvement for MRC is given by $\bar{\gamma}_{MRC} = \sum_{k=1}^{M} \bar{\gamma}_k = M\gamma_0$

No of Branches (M) $\gamma_0 = -20dB$	SC $\bar{\gamma}_{SC} = \gamma_0 \sum_{k=1}^{M} \frac{1}{K}$	MRC $\bar{\gamma}_{MRC} = M\gamma_0$
1	-20	-20
2	-18.2	-17
4	-16.8	-14
6	-16.1	-12.2

Space (Antenna) Diversity

Example: 1

A service provider wants to deploy diversity to improve received SNR such that the output instantaneous SNR is greater than average SNR γ_0, for 80% of the time. Determine the minimum number of diversity branches to be implemented to achieve this, assuming selection combining method.

Solution

- For SC: $p(\gamma_1, ..., \gamma_M \leq \gamma) = [1 - e^{-\frac{\gamma}{\gamma_0}}]^M$
- Thus $p(\gamma_1, ..., \gamma_M > \gamma) = 1 - [1 - e^{-\frac{\gamma}{\gamma_0}}]^M = 0.8$
- Hence $M = 4$.

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Diversity Techniques

Space (Antenna) Diversity

Example: 2

Compute the SNR improvement of 2-branch and 6-branch selection diversity over the case of no diversity.

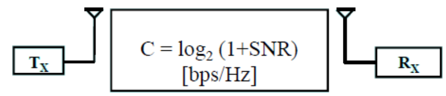
Solution

- No diversity:
  SNR improvement $= 10\log_{10}(\gamma_0/\gamma_0) = 0$ dB
- 2-branch selection diversity:
  SNR improvement $= 10\log_{10}(\bar{\gamma}_{SC}/\gamma_0) = 10\log_{10}(\gamma_0 \sum_k (1/k)/\gamma_0)$
  $= 10\log_{10} \sum_{k=1}^{2} \frac{1}{k} = 10\log_{10}(1.5) \approx 1.8$ dB
- 6-branch selection diversity:
  SNR improvement $= 10\log_{10} \sum_{k=1}^{6} \frac{1}{k} = 10\log_{10}(2.45) \approx 3.9$ dB
- Best performance/complexity ratio: 2-branch.
- Therefore, 2G/3G have implemented 2-branch diversity.

Capacity of fading channels with diversity

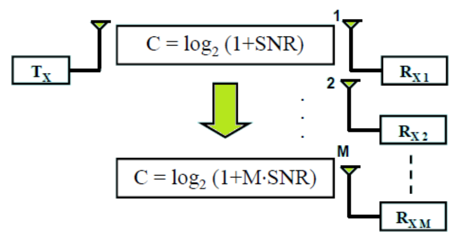
SISO System

- Capacity is limited by SNR. Each extra bps/Hz requires roughly doubling of the Tx power i.e. $\log_2(2SNR) = \log_2(2) + \log_2(SNR)$
- To go from 1bps/Hz to 11 bps/Hz, the Tx power must be increased by $\approx$ 1000 times!.



SIMO System (Capacity of receiver-side Diversity System:)

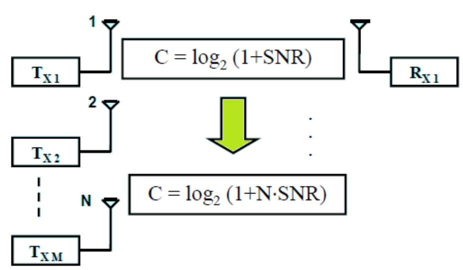
- Adding an antenna array at the receiver, known as RX diversity, provide diversity against fading or better BER.
- But it does not change the (slow growth) logarithmic nature of the bandwidth efficiency limit (or capacity).



Capacity of fading channels with diversity

MISO System (Capacity of transmitter-side Diversity System)

- Adding an array at the transmitter provide diversity against fading, but it does not change the (slow growth) logarithmic nature of the bandwidth efficiency limit (or capacity).

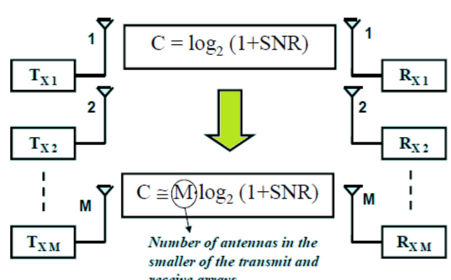


MIMO System

- Spatial Multiplexing: incoming bits are partitioned into N sub-streams, and each transmuted in parallel on different antenna. No diversity gain.
- Diversity: Provides link quality improvements. coding and diversity gain may be achieved.

MIMO System (Capacity of MIMO-MUX System)

- With dual arrays, the bandwidth efficiency growth is linear with the number of antennas. This is in contrast with the logarithmic growth obtained with a single array (conventional diversity).



Channel and Speech Coding for mobile radio

Speech (source) Coding

It removes redundant bits, to reduce data size.

Channel Coding

- It adds new redundant bits to protect data against fading effects of the channel.
- Channel coding combined with modulation can be used to approach channel capacity: $C = \log_2(1 + SNR)$ bits/sec/Hz.
- Channel coding compulsory in cellular system to combat small-scale fading.

Channel Coding types

- Block codes: Hamming code, Golay code, BCH, Reed Solomon codes. it is used mostly in non real-time applications: deep space communication, military communications, cellular digital packet data. Solomon codes.
- Convolutional Codes: Trellis and Turbo codes. widely used in 2G/3G cellular